# Derivations of Speaker Response, Sealed and Ported

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As part of my electronics hobby, I decided to try building my own speakers for electric and upright bass. There are freeware design programs out there such as WinISD and UniBox. If you just want to design speakers, this document won't help at all.

Instead, the purpose of this report is to summarize the basic physics of sealed and ported speakers, with derivations of the formulas. A secondary purpose is to de-mystify the inner workings of speakers.

I wrote a companion spreadsheet for this document, which uses the derived formulas to graph the most important speaker analyses. It is not so much for use as a design program (there are better ones available for free) but as a way to check my results against the free programs.

# References

There are a couple of nice web pages along the lines of my derivation. I took material as needed from these pages.

http://www.arcavia.com/kyle/Equations/index.html
http://www.silcom.com/~aludwig/
http://www.diysubwoofers.org/

The formatting of this document is due to its origin as a workbook from a math program called MuPad. In addition, I have an Excel spreadsheet at my website, which implements the formulas derived here.

# General Strategy

I think that an overview can help a lot in understanding where I am going with these calculations. The ultimate goal is to generate the most important curves used in DIY speaker design: Sensitivity, cone excursion, port air speed. My approach is along these lines:

1. How a driver works -- electromechanical parameters defined.

2. Thiele-Small parameters defined for the purpose of using published driver data.

3. Equation of motion and its solution -- identifying the forces on the speaker cone, and solving Newton's F=ma to obtain the basic excursion versus frequency function. Everything that follows beyond this point is derived as an extension to the excursion function for the driver.

4. Cone excursion versus frequency in sealed box -- the box is modeled as a simple modification to the spring constant of the driver. The "effective spring constant" is how I deal with both sealed and ported boxes.

5. Cone excursion versus frequency in ported box -- we come up with an effective spring "constant" for the ported box, which is a frequency-dependent function. It is worth noting that we use this one function for everything else, mainly to avoid making mistakes in subsequent calculations.

6. Sensitivity -- turning cone motion into sound pressure.

7. Port air speed -- the cone and port motion are coupled, so we use the cone motion formula to compute the port air speed.

8. Distortion -- borrowing a derivation from Kyle Lahnkoski’s web page, we look at how the finite size of the box causes harmonic distortion.

9. Transient response -- a Fourier Transform technique for computing the transient response of the cone based on the cone excursion versus frequency function

10. Near-field measurement technique -- the simple coupling between cone and port allows a ported system response function to be computed from a near-field measurement of cone response. The result allows DIY’ers to measure the response of ported speakers without an anechoic chamber.

11. Impedance -- this will be brief, as I have not found a huge use for the impedance graph.

# 1. How a driver works

My model of the driver is based on a simple list of electromechanical parameters, which completely describe the behavior of the system at frequencies where the cone is acting as a simple piston.

X = Excursion of cone, i.e., distance from the resting position

M = Cone mass.

Kdriver = Spring constant of driver. It is the proportionality constant in Hooke’s Law.

b = Mechanical damping. This is the proportionality constant for a law describing a "dragging" or frictional force proportional to cone velocity.

A = Cone area.

BL = Product of voice coil wire length and magnetic field. These two factors always go together, so they are treated as a single symbol.

R = Resistance of voice coil wire.

L = Inductance of voice coil.

V = Input voltage.

# 2. Thiele-Small parameters defined

These are simply taken from Kyle Lahnkoski's web page. My approach is to convert the T-S parameters directly into electromechanical parameters, and use the latter numbers in all subsequent calculations. But you have to deal with the T-S parameters, because they are what the driver makers typically provide. We are given the following parameters:

Fs = Driver resonant frequency in Hz

Vas = Sealed box with same effective spring constant as driver

Sd = Area of driver, same as A in my equations

Qms = Mechanical "Q" of driver

Qes = Electrical "Q" of driver

R = Electrical resistance of coil

L = Inductance of coil

I use the following conversions:

```
• "Driver resonant frequency in radial units", Ws = 2*PI*Fs;
  "Spring constant of driver", Kdriver = 1.4*Patm*Sd^2/Vas;
  "Damping constant of driver", b = Kdriver/Qms/Ws;
  "Mass of driver", M = Kdriver/Ws^2;
  "Field length product", BL = sqrt(Kdriver/Qes/Ws*R);
```

"Driver resonant frequency in radial units", $\mathrm{Ws} = 2 \cdot \pi \cdot \mathrm{Fs}$

"Spring constant of driver", $\mathrm{Kdriver} = \dfrac{1.4 \cdot \mathrm{Sd}^2 \cdot \mathrm{Patm}}{\mathrm{Vas}}$

"Damping constant of driver", $b = \dfrac{\mathrm{Kdriver}}{\mathrm{Ws} \cdot \mathrm{Qms}}$

"Mass of driver", $M = \dfrac{\mathrm{Kdriver}}{\mathrm{Ws}^2}$

"Field length product", $\mathrm{BL} = \sqrt{\dfrac{R \cdot \mathrm{Kdriver}}{\mathrm{Ws} \cdot \mathrm{Qes}}}$

In addition, some physical constants are helpful:

```
• "Atmospheric pressure", Patm = 101.3e3 *Pa;
  "Density of air", Rho = 1.18*kg/m^3;
  "Speed of sound", C = 345*m/s;
  "0 dB reference for sound pressure", SplRef = 2e-5*Pa;
```

"Atmospheric pressure", $\mathrm{Patm} = 101300.0 \cdot \mathrm{Pa}$

"Density of air", $\text{Rho} = \frac{1.18 \cdot \text{kg}}{m^3}$

"Speed of sound", $C = \frac{345 \cdot m}{s}$

"0 dB reference for sound pressure", $\text{SplRef} = 0.00002 \cdot \text{Pa}$

# 3. Equation of motion and its solution

The strategy is to list all of the forces on the cone, so we can apply F=ma. First I will look at the electrical side of things:

The voltage V across the wire is the sum of the voltage drop across series resistance R and the "back EMF" induced by the motion of the voice coil in the magnetic field:

- `V = Iin*R + X'*BL`

  $$V = \text{BL} \cdot X' + R \cdot \text{Iin}$$

Please note that we are neglecting inductance for now. We want to do everything with input voltage, not current, since an amplifier is a voltage source. But the force on the cone is proportional to the current, so we have to do some manipulations. Solving for the voice coil current,

- `Iin=(V - X'*BL)/R`

  $$\text{Iin} = \frac{V - \text{BL} \cdot X'}{R}$$

Note that X' is the time derivative of X, i.e., the velocity. We will also see that X" is the acceleration. The magnetic force on the voice coil is:

- `Fmag = Iin*BL;`
  `Fmag = BL*V/R - X'*BL^2/R`

  $$\text{Fmag} = \text{BL} \cdot \text{Iin}$$

  $$\text{Fmag} = \frac{V \cdot \text{BL}}{R} - \frac{\text{BL}^2 \cdot X'}{R}$$

Now we have an equation for the magnetic force, purely in terms of the input voltage.

Note that the second term in the magnetic force is a resistive or "damping" force that opposes the motion of the cone. *The magnetic field is doing two apparently opposing things in a speaker*, which is why you can't just arbitrarily increase the magnetic field.

Next we find the purely mechanical forces:

- `"Spring force on cone", Fspring = -Kdriver*X;`

```
"Mechanical damping force", Fdamping = -b*X';
"Inertial force", Finertia = -M*X'';
```

$$\text{"Spring force on cone", Fspring} = -X \cdot \text{Kdriver}$$

$$\text{"Mechanical damping force", Fdamping} = -b \cdot X'$$

$$\text{"Inertial force", Finertia} = -M \cdot X''$$

I am treating inertia, the right hand side of F=ma, as a force. In this parlance, F=ma is written by setting the sum of all forces to zero:

- `BL*V/R - X'*BL^2/R - M*X'' - b*X' - Kdriver*X = 0`

$$\frac{V \cdot \text{BL}}{R} - M \cdot X'' - X \cdot \text{Kdriver} - b \cdot X' - \frac{\text{BL}^2 \cdot X'}{R} = 0$$

This is everything. It's a differential equation with X as the unknown. Rearranging it into a standard form...

- `M*X'' + (BL^2/R + b)*X' + Kdriver*X = BL*V/R`

$$M \cdot X'' + X \cdot \text{Kdriver} + X' \cdot \left(b + \frac{\text{BL}^2}{R}\right) = \frac{V \cdot \text{BL}}{R}$$

This is the classic "damped, driven harmonic oscillator" equation of motion from kindergarten physics. To proceed, it is necessary to introduce complex numbers. We are going to use the familiar transformation to convert differential equations into algebraic equations in complex number space, where w is the angular frequency corresponding to frequency f:

- ```
  x = x;
  x' = I*w*x;
  x'' = -w^2*x;
  w = 2*PI*f;
  ```

$$x = x$$

$$x' = \mathrm{i} \cdot w \cdot x$$

$$x'' = -w^2 \cdot x$$

$$w = 2 \cdot f \cdot \pi$$

The equation of motion becomes:

- `-w^2*M*X + I*w*(BL^2/R + b)*X + Kdriver*X = BL*V/R`

$$X \cdot \text{Kdriver} - M \cdot X \cdot w^2 + \mathrm{i} \cdot X \cdot w \cdot \left(b + \frac{\text{BL}^2}{R}\right) = \frac{V \cdot \text{BL}}{R}$$
  ```

How easy to solve for X:

- X = BL*V/M/R/(Kdriver/M + I*w/M*(BL^2/R + b) - w^2)

$$X = \frac{V \cdot \mathrm{BL}}{M \cdot R \cdot \left(\frac{\mathrm{Kdriver}}{M} - w^2 + \frac{i \cdot w \cdot \left(b + \frac{\mathrm{BL}^2}{R}\right)}{M}\right)}$$

What a neat result. We have an exact formula for the displacement of the cone as a function of input voltage amplitude and frequency. For the closed box system, only one additional modification is needed, to account for voice coil inductance. We make the transformation:

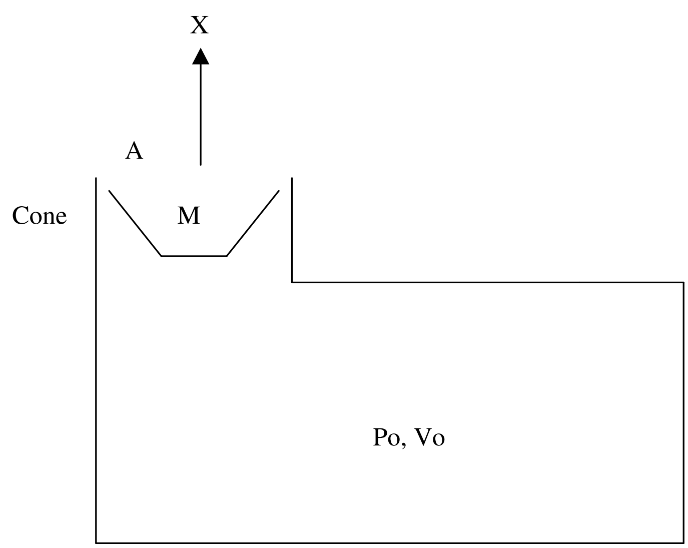
- Zcoil = R + I*w*Lcoil;
  X = BL*V/M/Zcoil/(Kdriver/M + I*w/M*(BL^2/Zcoil + b) - w^2)

$$\mathrm{Zcoil} = R + i \cdot w \cdot \mathrm{Lcoil}$$

$$X = \frac{V \cdot \mathrm{BL}}{M \cdot \mathrm{Zcoil} \cdot \left(\frac{\mathrm{Kdriver}}{M} - w^2 + \frac{i \cdot w \cdot \left(b + \frac{\mathrm{BL}^2}{\mathrm{Zcoil}}\right)}{M}\right)}$$

This is the "master" response function for everything that follows.

# 4. Cone excursion vs frequency in sealed box




The cone has mass M and frontal area A. Its excursion is X. The driver is loaded into a box with internal air volume Vo, at atmospheric pressure Po.

As the volume changes, so does the pressure. The relationship is Boyle’s Law. (I am cheating a bit here. You should use not atmospheric pressure, but the adiabatic bulk modulus, which is 1.4 times atmospheric pressure. My worked example takes this factor into account.)

- `P*V = 1.4*Patm*Vbox`

  $$P \cdot V = 1.4 \cdot \mathrm{Patm} \cdot \mathrm{Vbox}$$

- `P=1.4*Patm*Vbox/V`

  $$P = \frac{1.4 \cdot \mathrm{Patm} \cdot \mathrm{Vbox}}{V}$$

A small change in volume, such as when the cone moves in and out, produces a small change in force due to the pressure difference on the two sides of the cone...

- `dP = diff(1.4*Patm*Vbox/V,V)*dV`

  $$\mathrm{dP} = -\frac{1.4 \cdot \mathrm{dV} \cdot \mathrm{Patm} \cdot \mathrm{Vbox}}{V^2}$$

But we are talking about small changes in volume, so Vo remains approximately equal to V...

- `dP = - dV*Vbox/V`

  $$\mathrm{dP} = -\frac{\mathrm{dV} \cdot \mathrm{Vbox}}{V}$$

The change in pressure results in a force on the cone proportional to the cone area:

- `dF = A*dP`

  $$\mathrm{dF} = A \cdot \mathrm{dP}$$

Beware that I am being a bit cavalier by making the differential symbols disappear, and replacing them with the absolute force and displacement. This is just a minor change of notation, as the force on the cone is really driven by the pressure difference inside the box and outside.

- `F = -A^2*1.4*Patm*X/Vbox`

  $$F = -\frac{1.4 \cdot A^2 \cdot X \cdot \mathrm{Patm}}{\mathrm{Vbox}}$$

A negative restoring force proportional to displacement looks like Hooke’s Law for a spring with a spring constant of Kbox.

- `F = -Kbox*X`

  $$F = -X \cdot \mathrm{Kbox}$$

The spring constant is:

- ```
  Kbox = A^2*1.4*Patm/Vbox
  ```

  $$\text{Kbox} = \frac{1.4 \cdot A^2 \cdot \text{Patm}}{\text{Vbox}}$$

This is a crucial result. *The box is a spring, and just a spring*. The spring constant depends on the driver size, but it can be adjusted by changing the volume of the box. *The box is an adjustable spring*.

To analyze the sealed system, one only needs to replace Kdriver with a new "total" spring constant:

- ```
  Ktot = Kdriver + Kbox
  ```

  $$\text{Ktot} = \text{Kbox} + \text{Kdriver}$$

The Thiele-Small parameters include a mysterious parameter Vas, which is simply the box volume with the same spring constant as the mechanical spring constant of the driver. Now that we understand the relationship between spring constant and box volume, we can use it to convert Vas into the driver spring constant.

- ```
  Kdriver = A^2*1.4*Po/Vas
  ```

  $$\text{Kdriver} = \frac{1.4 \cdot A^2 \cdot \text{Po}}{\text{Vas}}$$

- ```
  p = A*rho*X''/4/PI/r;
  p = w^2*rho*A/4/PI/r;
  ```

  $$p = \frac{A \cdot X'' \cdot \text{rho}}{4 \cdot r \cdot \pi}$$

  $$p = \frac{A \cdot w^2 \cdot \text{rho}}{4 \cdot r \cdot \pi}$$

With rho being the density of air, 1.2 kg/m^3. The factor of frequency squared is really interesting. We can make some extrapolations of the equation for X at high frequency. If w gets large enough, then it dominates the denominator, and:

- ```
  X = BL*V/m/R/w^2;
  p = BL*V/m/R*rho*A/4/PI/r
  ```

  $$X = \frac{V \cdot \text{BL}}{R \cdot m \cdot w^2}$$

  $$p = \frac{A \cdot V \cdot \text{BL} \cdot \text{rho}}{4 \cdot R \cdot m \cdot r \cdot \pi}$$

The last formula is why speakers work so well for being so simple. It means that an ideal cone driver has utterly flat frequency response at higher frequencies. Thus,
  ```

controlling the high frequency behavior of cone speakers is a matter of dealing with deviations from ideal behavior.

When I use the above formula, with the standard SPL reference pressure of 20 micro-pascals, my curves are all roughly 2 dB below the results from programs such as WinISD and UniBox. For this reason, I am actually using the following formulation instead: The acoustical output power of the speaker is documented by Kyle Lahnakoski to be:

- `P = Rho*Sd^2*abs(X'')^2/2/PI/C`

$$P = \frac{Sd^2 \cdot Rho \cdot |X''|^2}{2 \cdot C \cdot \pi}$$

Here, C is the speed of sound in air. This is the output power of the speaker. The law of energy conservation dictates that the same amount of power passes through a sphere of radius r, so that the acoustical power per unit area is the output power divided by the area of the sphere:

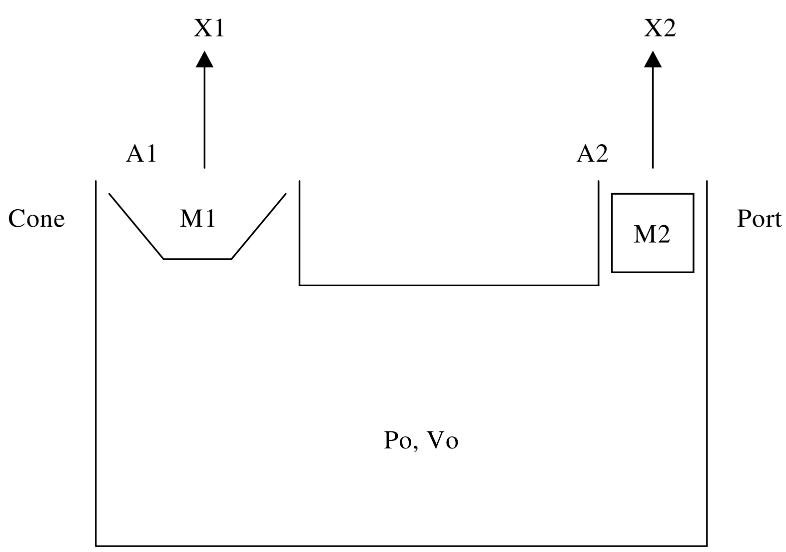
- `W = P/4/PI/r^2;`
  `W = 2*Rho/C*(Sd*abs(X'')/4/PI/r)^2`

$$W = \frac{P}{4 \cdot r^2 \cdot \pi}$$

$$W = \frac{Sd^2 \cdot Rho \cdot |X''|^2}{8 \cdot C \cdot r^2 \cdot \pi^2}$$

And then, using 1 picowatt per square meter as the reference, I get the same SPL curves as the design software puts out.

# 5. Ported system



The ported system has the same cone with mass M1, frontal area A1, and excursion X1. The secret of the port is disarmingly simple: *The "plug" of air inside the port gets blown in and out by the pressure in the box. Its motion produces sound.* By sharing the same box volume with the cone, the two become coupled to one another. Here we label the mass of air in the port as M2, its frontal area A2, and its excursion X2.

Going back to the result from Boyle's law that is given above, now we have two pistons adding to the pressure and volume in the box. The change in total volume is now:

- `dV = A1*X1 + A2*X2`

$$dV = A1 \cdot X1 + A2 \cdot X2$$

Thus the change in pressure looks like:

- `dP = -1.4*Patm/Vbox*(A1*X1 + A2*X2)`

$$dP = -\frac{1.4 \cdot Patm \cdot (A1 \cdot X1 + A2 \cdot X2)}{Vbox}$$

Both ports are looking at the same pressure in the box. That's how they are coupled. This pressure creates forces on the cone:

- `F1 = A1*dP;`
  `F1 = -1.4*Patm/Vbox*A1*(A1*X1 + A2*X2)`

$$F1 = A1 \cdot dP$$

$$F1 = -\frac{1.4 \cdot A1 \cdot Patm \cdot (A1 \cdot X1 + A2 \cdot X2)}{Vbox}$$

I am assuming that the cone is driven, so that F1 is just one of the many forces added up according to Newton's second law. The port is free to move, but its mass imposes an inertial force, and the sum of forces is zero:

- `F2 = A2*dP - M2*X2'';`
  `F2 = 0`

$$F2 = A2 \cdot dP - M2 \cdot X2''$$

$$F2 = 0$$

We switch into complex space, and write the equation of motion for the port:

- `-1.4*Patm*A2/Vbox*(A1*X1 + A2*X2) + M2*w^2*X2 = 0`

$$w^2 \cdot M2 \cdot X2 - \frac{1.4 \cdot A2 \cdot Patm \cdot (A1 \cdot X1 + A2 \cdot X2)}{Vbox} = 0$$

Okay, now we can treat the equations for F1 and F2 as two linear equations in two unknowns -- X1 and X2. The next step is to eliminate X2, so we have an equation of F1 that is solely dependent on X1. I don't know if this is the best point to introduce a symbol, but I will define the resonant frequency of the port in angular units:

```
• Kport = A2^2*1.4*Patm/Vbox;
  Wport = sqrt(Kport/M2)
```

$$\text{Kport} = \frac{1.4 \cdot \text{A2}^2 \cdot \text{Patm}}{\text{Vbox}}$$

$$\text{Wport} = \sqrt{\frac{\text{Kport}}{\text{M2}}}$$

The rest of the derivation...

```
• -1.4*Patm*A2/Vbox*(A1*X1 + A2*X2) +
     A2^2*1.4*Patm*w^2*X2/Wport^2*Vbox = 0;
  1.4*Patm*A1*A2*X1/Vbox = X2*(A2^2*1.4*Patm*w^2/Wport^2/Vbox
     - 1.4*Patm*A2^2/Vbox);
  A2*X2 = A1*X1/(w^2/Wport^2-1);
  F1 = -1.4*Patm*A1/Vbox*(A1*X1 + A1*X1/(w^2/Wport^2-1));
  F1 = -1.4*Patm*A1^2*X1/Vbox*(1+1/(w^2/Wport^2-1));
  F1 = -Kbox*X1*(1+1/(w^2/Wport^2-1))
```

$$\frac{1.4 \cdot \text{A2}^2 \cdot w^2 \cdot \text{X2} \cdot \text{Patm} \cdot \text{Vbox}}{\text{Wport}^2} - \frac{1.4 \cdot \text{A2} \cdot \text{Patm} \cdot (\text{A1} \cdot \text{X1} + \text{A2} \cdot \text{X2})}{\text{Vbox}} = 0$$

$$\frac{1.4 \cdot \text{A1} \cdot \text{A2} \cdot \text{X1} \cdot \text{Patm}}{\text{Vbox}} = \text{X2} \cdot \left( \frac{1.4 \cdot \text{A2}^2 \cdot w^2 \cdot \text{Patm}}{\text{Vbox} \cdot \text{Wport}^2} - \frac{1.4 \cdot \text{A2}^2 \cdot \text{Patm}}{\text{Vbox}} \right)$$

$$\text{A2} \cdot \text{X2} = \frac{\text{A1} \cdot \text{X1}}{\frac{w^2}{\text{Wport}^2} - 1}$$

$$\text{F1} = -\frac{1.4 \cdot \text{A1} \cdot \text{Patm} \cdot \left( \text{A1} \cdot \text{X1} + \frac{\text{A1} \cdot \text{X1}}{\frac{w^2}{\text{Wport}^2} - 1} \right)}{\text{Vbox}}$$

$$\text{F1} = -\frac{1.4 \cdot \text{A1}^2 \cdot \text{X1} \cdot \text{Patm} \cdot \left( \frac{1}{\frac{w^2}{\text{Wport}^2} - 1} + 1 \right)}{\text{Vbox}}$$

$$\text{F1} = -\text{X1} \cdot \text{Kbox} \cdot \left( \frac{1}{\frac{w^2}{\text{Wport}^2} - 1} + 1 \right)$$

This is exactly a linear restoring force -- a spring -- but with a frequency dependent spring constant. Also, the factor in front of the parentheses is exactly the old box spring constant for the ported system. Now we can write:

- `Ksystem = Kbox*(1+1/(w^2/Wport^2-1))`

$$\text{Ksystem} = \text{Kbox} \cdot \left( \frac{1}{\frac{w^2}{\text{Wport}^2} - 1} + 1 \right)$$

At the port resonance, the spring constant becomes infinite. (In reality, it remains finite because the port experiences some damping). This confirms the widely noted theory that at port resonance, the cone grinds to a halt, and the port is producing all of the sound. All we need to do in order to model the excursion of the cone is to substitute this value of the spring constant into the closed box excursion formula given above. Easy, eh? The last step is to sum the acoustical output of the cone and the port. The total volume displacement:

- `dVtot = A1*X1 + A2*X2;`
  `dVtot = A1*X1*(1+1/(w^2/Wport^2-1))`

$$\text{dVtot} = \text{A1} \cdot \text{X1} + \text{A2} \cdot \text{X2}$$

$$\text{dVtot} = \text{A1} \cdot \text{X1} \cdot \left( \frac{1}{\frac{w^2}{\text{Wport}^2} - 1} + 1 \right)$$

Thus, to predict the acoustical output, we replace the cone area A1 in the closed box formulas with a frequency dependent effective cone area:

- `Aeff = A1*(1+1/(w^2/Wport^2-1))`

$$\text{Aeff} = \text{A1} \cdot \left( \frac{1}{\frac{w^2}{\text{Wport}^2} - 1} + 1 \right)$$

# 6. Sensitivity

I lost the reference for this formula, but am using the following technique right now. Sound pressure should be proportional to the volume acceleration of the cone. Thus I end up with the formula:

- `P = Aeff*w^2*X*rho/2/PI/r`

$$P = \frac{X \cdot w^2 \cdot \text{rho} \cdot \text{Aeff}}{2 \cdot r \cdot \pi \cdot \text{Pr}}$$

Aeff = Effective driver area as given above
w = Angular frequency. Its square converts excursion into acceleration
X = RMS cone excursion
rho = Density of air
r = Distance of listener away from speaker

Thus the sound pressure level (SPL) is obtained by:

- `SPL = 20*log(10,P/Pr)`

  $$\mathrm{SPL} = 20 \cdot \log_{10}\left(\frac{P}{\mathrm{Pr}}\right)$$

  Pr = Reference pressure, equal to 20 µPa.

Bear with me while I track down the reference for this.

# 7. Port air speed

I take cone excursion as the "master" function, from which I derive everything else. As it turns out, port and cone motion are coupled together, so you can compute one from the other. The port air speed formula is:

- `v = w*X*A1/A2*(w^2/wport^2-1)`

  $$v = \frac{X \cdot \mathrm{A1} \cdot w \cdot \left(\frac{w^2}{\mathrm{wport}^2} - 1\right)}{\mathrm{A2}}$$

The reason for plotting port air speed is to make sure it never gets to be an appreciable fraction of the speed of sound, which would result in audible turbulence distortion. Thus it is convenient to graph port air speed in Mach units:

- `Mach = v/c`

  $$\mathrm{Mach} = \frac{v}{c}$$

  c = 345 m/s, the speed of sound

# 8. Distortion

This is where I take a simplistic formula from Kyle Lahnkoski's web page. So far, we have discussed a system that is totally linear. But a number of distortion effects are possible:

a. Magnetic nonlinearity, meaning that the magnetic field is not exactly constant as the cone moves in and out. This nonlinearity can be appreciable, as the Xmax parameter for cone excursion is typically specified as the point where the driver reaches 10% THD.

b. Suspension nonlinearity, meaning that the cone suspension (surround and spider) are not ideal springs.

c. Pressure nonlinearity. Looking back at Boyle's Law, it is not utterly linear. This is what I am interested in graphing, as it is potentially an effect of small boxes. Here, I take the formula directly from Kyle's page:

- ```
  THD = (y+1)/y*(1-1/(1+Aeff*X/Vbox)^y);
  y = 1.4;
  ```
  ```

$$\text{THD} = \frac{(y+1)\cdot\left(1-\frac{1}{\left(\frac{X\cdot \text{Aeff}}{\text{Vbox}}+1\right)^{y}}\right)}{y}$$

$$y = 1.4$$

# 9. Transient response

There is a lot of discussion and marketing hype surrounding transient response of speakers. But I have never seen any information relating transient response and how speakers actually sound. Still, I got curious about plotting the transient response curve of a speaker. Since I am computing the frequency response, I decided to look for a transform method. One interpretation of the (complex) frequency response curve is that it is the Fourier transform of the transient response function for an infinite impulse input. My reference is actually from optics:

> http://lw.pennnet.com/Articles/Article_Display.cfm
> ?Section=Articles&Subsection=Display&ARTICLE_ID=152437

The following quite captures the essence:

> Then the windowed data is inverse Fourier transformed into the time domain. To compute the TDR step response, a 1/jw scaling is needed before the inverse Fourier transform, because 1/jw is the Fourier transform of a step response:

So far, so good. The transient response curve for any input can be found by convoluting the infinite impulse response function with the input function. But the Convolution Theorem says that convolution in time space is the same as multiplication in frequency space. So my scheme for finding the step function response goes like this:

1. Find the Fourier Transform of the step function. It is equal to 1/jw.

2. Multiply by the complex response curve for acceleration versus frequency.

3. Take the Fourier Transform of the product.

4. Extract the transient response curve.

More detailed notes will be added later. The best thing to do is look at my spreadsheet, which computes the transient response curve for a ported system.

# 10. Near-field measurement technique

I am of the opinion that measurement is unnecessary for DIY speakers that are carefully built from good components. The speakers I have built conform faithfully to the basic model. But there are other "unknowns" whose performance might be interesting to measure, including preamp voicing and the response curves of both good and bad commercial speakers. To this end, I designed a real time analyzer program, then came up against my lack of an anechoic chamber. So I developed a technique for measuring the response of ported speakers using a near-field measurement. The steps are:

1. Place a measurement microphone directly in front of the speaker cone, so it "hears" mostly the cone with minimal contribution from the port.

2. Measure the near-field response curve of the driver.

3. Looking at the curve, estimate the port tuning frequency from the "dip" in response.

4. Correct the near-field response curve with the following function:

```
dBsystem = dBdriver + 20*log(10,abs((1+1/(w^2/Wport^2-1))))
```

$$\mathrm{dBsystem} = \mathrm{dBdriver} + 20 \cdot \log_{10}\left(\left|\frac{1}{\frac{w^2}{\mathrm{Wport}^2} - 1} + 1\right|\right)$$

# 11. Impedance

To be honest I have not had much use for the impedance graph, but it's only fair to include it in this treatment, since it is a common calculation. Let's look back at what we've already got. Remember this relationship

```
Iin = (V - X'*BL)/R
```

$$\mathrm{Iin} = \frac{V - \mathrm{BL} \cdot X'}{R}$$

Okay, so we've got current, voltage, and excursion. Recalling the phasor transformation, we can combine these equations:

```
X := BL*V/M/Zcoil/(Kdriver/M + I*w/M*(BL^2/Zcoil + b) - w^2);
Iin := collect((V - I*w*X*BL)/R,V);
```

$$\frac{V \cdot \mathrm{BL}}{M \cdot \mathrm{Zcoil} \cdot \left(\frac{\mathrm{Kdriver}}{M} - w^2 + \frac{\mathrm{i} \cdot w \cdot \left(b + \frac{\mathrm{BL}^2}{\mathrm{Zcoil}}\right)}{M}\right)}$$

$$\frac{V \cdot \left(1 - \frac{\mathrm{i} \cdot w \cdot \mathrm{BL}^2}{M \cdot \mathrm{Zcoil} \cdot \left(\frac{\mathrm{Kdriver}}{M} - w^2 + \frac{\mathrm{i} \cdot w \cdot \left(b + \frac{\mathrm{BL}^2}{\mathrm{Zcoil}}\right)}{M}\right)}\right)}{R}$$

Finally, impedance is the ratio of voltage to current:

```
Z = V/Iin
```

$$Z = \frac{R}{1 - \frac{i \cdot w \cdot BL^2}{M \cdot Zcoil \cdot \left( \frac{Kdriver}{M} - w^2 + \frac{i \cdot w \cdot \left( b + \frac{BL^2}{Zcoil} \right)}{M} \right)}}$$

This is the equation for the "raw" driver. For the system impedance, you need to perform the modifications to the spring constant as described above.