Speaker frequency and phase response - joined at the hip?

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Start with the reciprocal of the complex expression for cone excursion, leaving out pure proportionality terms. This is from my little paper:

$$R \cdot \left(\frac{\mathrm{Ktot}}{m} - w^2 + \frac{i \cdot w \cdot \left(b + \frac{\mathrm{BL}^2}{R} \right)}{m} \right)$$

The strategy is to find a function for the *empirical* response curve, whose number of parameters we can reliably determine. A polynomial in w, with constant coefficients, would be such a function. Looking ahead, it is important to expand the above expression to get R out of the denominator:

$$\frac{i \cdot R \cdot b \cdot w}{m} - R \cdot w^2 + \frac{R \cdot \mathrm{Ktot}}{m} + \frac{i \cdot w \cdot \mathrm{BL}^2}{m}$$

As per the paper, R is replaced with the correct complex coil reactance based on DC resistance Rcoil and inductance L.

$$\frac{i \cdot w \cdot \mathrm{BL}^2}{m} - w^2 \cdot (i \cdot L \cdot w + \mathrm{Rcoil}) + \frac{i \cdot b \cdot w \cdot (i \cdot L \cdot w + \mathrm{Rcoil})}{m} + \frac{\mathrm{Ktot} \cdot (i \cdot L \cdot w + \mathrm{Rcoil})}{m}$$

Use the math tool to expand this expression and collect terms in w.

$$\frac{\mathrm{Ktot} \cdot \mathrm{Rcoil}}{m} - i \cdot L \cdot w^3 + w^2 \cdot \left(-\mathrm{Rcoil} - \frac{L \cdot b}{m} \right) + w \cdot \left(\frac{i \cdot L \cdot \mathrm{Ktot}}{m} + \frac{i \cdot b \cdot \mathrm{Rcoil}}{m} + \frac{i \cdot \mathrm{BL}^2}{m} \right)$$

Now we have a useful result for sealed speakers, where Ktot is a simple constant equal to the sum of the driver and box spring constants. It is a 3rd order polynomial in w, and the coefficients are all constants, so the *empirical* complex response curve has exactly four parameters. We also think that the measured curve has at least four parameters (it had better not have more):

1. Overall sensitivity, i.e. a proportionality factor
2. Box tuning frequency
3. Total Q
4. Coil inductance

Thus, the complex and amplitude curves have the same number of parameters – no information is lost by taking the absolute value.

Now we turn to the ported system. Here you must forgive me for the small type, as my math program just lays everything out in a single line, and then Word turns it into a bitmap. Computers.

Following my own instructions, we replace the sealed box spring constant with a frequency dependent spring constant. We are just looking at cone motion right now:

$$\mathrm{Ktot} = \frac{w^2 \cdot (\mathrm{Kbox} + \mathrm{Kdriver})}{w^2 - \mathrm{wPort}^2}$$

Now the reciprocal of cone amplitude gets pretty ugly, but we will clean it up a bit.

$$w^2 \cdot \left(-\mathrm{Rcoil} - \frac{L \cdot b}{m}\right) - \mathrm{i} \cdot L \cdot w^3 + \frac{w^2 \cdot \mathrm{Rcoil} \cdot (\mathrm{Kbox} + \mathrm{Kdriver})}{m \cdot \left(w^2 - \mathrm{wPort}^2\right)} + w \cdot \left(\frac{\mathrm{i} \cdot b \cdot \mathrm{Rcoil}}{m} + \frac{\mathrm{i} \cdot \mathrm{BL}^2}{m} + \frac{\mathrm{i} \cdot L \cdot w^2 \cdot (\mathrm{Kbox} + \mathrm{Kdriver})}{m \cdot \left(w^2 - \mathrm{wPort}^2\right)}\right)$$

What's ugly is that there is stuff in the denominator, and my brain can only comprehend something simple like a polynomial, so I will try to get rid of denominators. Expanding the above:

$$\frac{\mathrm{i} \cdot b \cdot w \cdot \mathrm{Rcoil}}{m} - w^2 \cdot \mathrm{Rcoil} - \mathrm{i} \cdot L \cdot w^3 + \frac{\mathrm{i} \cdot w \cdot \mathrm{BL}^2}{m} - \frac{L \cdot b \cdot w^2}{m} + \frac{\mathrm{i} \cdot L \cdot w^3 \cdot \mathrm{Kbox}}{m \cdot \left(w^2 - \mathrm{wPort}^2\right)} + \frac{\mathrm{i} \cdot L \cdot w^3 \cdot \mathrm{Kdriver}}{m \cdot \left(w^2 - \mathrm{wPort}^2\right)} + \frac{w^2 \cdot \mathrm{Kbox} \cdot \mathrm{Rcoil}}{m \cdot \left(w^2 - \mathrm{wPort}^2\right)} + \frac{w^2 \cdot \mathrm{Rcoil} \cdot \mathrm{Kdriver}}{m \cdot \left(w^2 - \mathrm{wPort}^2\right)}$$

Finally get it all over a common denominator, and prepare to give up hope:

$$\frac{\mathrm{i} \cdot L \cdot m \cdot w^5 + w^4 \cdot (L \cdot b + m \cdot \mathrm{Rcoil}) + w^2 \cdot \left(-\mathrm{Kbox} \cdot \mathrm{Rcoil} - \mathrm{Rcoil} \cdot \mathrm{Kdriver} - L \cdot b \cdot \mathrm{wPort}^2 - m \cdot \mathrm{Rcoil} \cdot \mathrm{wPort}^2\right) + w \cdot \left(\mathrm{i} \cdot b \cdot \mathrm{Rcoil} \cdot \mathrm{wPort}^2 + \mathrm{i} \cdot \mathrm{BL}^2 \cdot \mathrm{wPort}^2\right) + w^3 \cdot \left(-\mathrm{i} \cdot L \cdot \mathrm{Kbox} - \mathrm{i} \cdot b \cdot \mathrm{Rcoil} - \mathrm{i} \cdot L \cdot \mathrm{Kdriver} - \mathrm{i} \cdot \mathrm{BL}^2 - \mathrm{i} \cdot L \cdot m \cdot \mathrm{wPort}^2\right)}{m \cdot \mathrm{wPort}^2 - m \cdot w^2}$$

The ugly denominator is recalcitrant. But wait. This is the cone amplitude, but (recalling the instructions in the paper), we can add the complex amplitudes of port and cone motion. The excursion is simply multiplied by:

$$\frac{w^2}{w^2 - \mathrm{wPort}^2}$$

What a lucky break. Goodbye denominator:

$$L \cdot b \cdot w^2 - \mathrm{i} \cdot b \cdot w \cdot \mathrm{Rcoil} - \mathrm{Kbox} \cdot \mathrm{Rcoil} - \mathrm{i} \cdot L \cdot w \cdot \mathrm{Kdriver} - \mathrm{Rcoil} \cdot \mathrm{Kdriver} - \mathrm{i} \cdot w \cdot \mathrm{BL}^2 - \mathrm{i} \cdot L \cdot w \cdot \mathrm{Kbox} + \mathrm{i} \cdot L \cdot m \cdot w^3 - L \cdot b \cdot \mathrm{wPort}^2 + m \cdot w^2 \cdot \mathrm{Rcoil} - \mathrm{i} \cdot L \cdot m \cdot w \cdot \mathrm{wPort}^2 - m \cdot \mathrm{Rcoil} \cdot \mathrm{wPort}^2 + \frac{\mathrm{i} \cdot b \cdot \mathrm{Rcoil} \cdot \mathrm{wPort}^2}{w} + \frac{\mathrm{i} \cdot \mathrm{BL}^2 \cdot \mathrm{wPort}^2}{w}$$

Finally, collect terms

$$\frac{\mathrm{i} \cdot L \cdot m \cdot w^4 + \mathrm{i} \cdot b \cdot \mathrm{Rcoil} \cdot \mathrm{wPort}^2 + w^3 \cdot (L \cdot b + m \cdot \mathrm{Rcoil}) + \mathrm{i} \cdot \mathrm{BL}^2 \cdot \mathrm{wPort}^2 + w \cdot \left(-\mathrm{Kbox} \cdot \mathrm{Rcoil} - \mathrm{Rcoil} \cdot \mathrm{Kdriver} - L \cdot b \cdot \mathrm{wPort}^2 - m \cdot \mathrm{Rcoil} \cdot \mathrm{wPort}^2\right) + w^2 \cdot \left(-\mathrm{i} \cdot L \cdot \mathrm{Kbox} - \mathrm{i} \cdot b \cdot \mathrm{Rcoil} - \mathrm{i} \cdot L \cdot \mathrm{Kdriver} - \mathrm{i} \cdot \mathrm{BL}^2 - \mathrm{i} \cdot L \cdot m \cdot \mathrm{wPort}^2\right)}{w}$$

If you have better eyesight than me, you can see that the numerator is a 4th order polynomial with constant coefficients. Thus the empirical complex function has exactly five parameters. We also think that the measured response curve of a ported speaker has five parameters:

1. Overall magnitude
2. Box tuning frequency
3. Port tuning frequency
4. Total Q
5. Coil inductance

Again, going from the complex to the magnitude curve, no information is lost. This is how I conclude that the amplitude and phase response curves are "joined at the hip."

Now, before you go thinking that I have mathematically proven a universal result, I will mention a serious practical problem with my method:

My model assumes that you have an accurate response curve at your disposal, with no noise, and going down to an arbitrarily low frequency. Especially in the case of subwoofers, this might not be possible. In particular, you need to go low enough to determine if the speaker has a 12 or 24 dB/oct cutoff. If you don't have ideal curves then the difference between two curves might be real but un-measurable. That's where two speakers with the "same" amplitude curves could still have different phase response curves.